[This question paper contains 7 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **1831** **GC-4**

Unique Paper Code : 32371208

Name of the Course : **B.Sc.(Hons.) Statistics**

Name of the Paper : Probability & Probability Distributions

Semester : II

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt **six** questions in **all**.

(c) Question No. **1** is compulsory. From the remaining questions do any **five** questions by selecting at least **two** questions from **each** section.

(d) Use of simple calculator is allwoed.

1. (a) For what value of k,

$$f(x) = \begin{cases} ke^{-2x} & ; \quad x \geq 0, \\ 0 & ; \quad x < 0, \end{cases}$$

is the probability density function of a random variable X.

(b) If X and Y are independent Cauchy variates with parameters (λ_1, μ_1) and (λ_2, μ_2) respectively then find the distribution of (X + Y). 1

(c) If the characteristic function $\varphi_x(t)$ of a continuous random variable X is given, then how will you find p.d.f f(x) ? 1

(d) State the relationship between $M_x(t)$ and μ'_r. 1

(e) If two variables X and Y are independent then what is E (Y/X) ? 1

(f) Let X be a random variable with probability distribution :

X = x	–2	2	4
P (X = x)	1/8	½	3/8

Find E(X) and Var (X) 2

(g) Let X and Y have the joint p.d.f.

$f(x, y) = c(2x + y)$; $0 \le x \le 1,\ 0 \le y \le 2$,

then find c. 2

(h) If Cov (aX + bY, bX + aY) = ab Var (X + Y), then comment on the independence of X and Y. 2

(i) The probability distribution of a random variable X is :

$P(X = x) = (1/2)^x$; x = 1, 2, ...

Then find mean and mode of X. 2

(j) If the p.d.f. of a random variable X is :

$f(x) = c \exp[-(x^2 - 6x + 9)/24]$; $-\infty < x < \infty$,

then find c, mean and variance of X. 2

Section – A

2. (a) Let X be a random variable with p.m.f. given by : 6

X = x	0	1	2	3	4	5	6	7
P(X = x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

(i) find k,

(ii) find P(X < 6) and P(0 < X < 5),

(iii) if $P(X \le a) > \frac{1}{2}$, find the minimum value of a,

(iv) determine the distribution function of X.

(b) For the distribution :

$$dF(x) = \begin{cases} y_0\left(1-\frac{1}{a}|x-b|\right)dx & , \quad b-a<x<b+a \\ 0 & , \quad \text{otherwise} \end{cases}$$

calculate y_0, mean and variance. 6

3. (a) Let X be a random variable taking non-negative integral values. If the moments of X are given by : 6

$E(X^r) = 0.6$; r = 1, 2, 3.........

then find m.g.f. of X. Also, show that $P(X = 0) = 0.4$, $P(X = 1) = 0.6$, $P(X \ge 2) = 0$.

(b) An urn contains balls numbered 1, 2, 3. First a ball is drawn from the urn and then a fair coin is tossed as many times, as the number shown on the ball drawn. Find the expected number of tails. 6

4. (a) Define the characteristics function of a random variable. Show that the characteristic function of sum of two independent random variables is equal to the product of their characteristic functions. Is the converse true, if not justify. 6

(b) If the joint p.m.f. of X and Y is given by : 6

$$p(x,y) = \frac{(x-y)^2}{7} \text{ for } x = 1, 2 \text{ and } y = 1, 2, 3.$$

find :

(i) the marginal distribution of Y,

(ii) the joint distribution of $U = X + Y$ and $V = X - Y$,

(iii) the marginal distribution of U.

Section – B

5. (a) Find the m.g.f. of standard binomial variate $(X-np)/\sqrt{npq}$ and obtain its limiting form as $n \to \infty$. Also interpret the result. 6

(b) Let X be the negative binomial variate with p.m.f.,

$$f(x)=\begin{cases}\binom{k+x-1}{x}q^x p^k & ; \quad x=0,1,2,\dots\\ 0 & ; \quad \text{otherwise}\end{cases}$$

Show that the moment recurrence formula is :

$$\mu_{r+1}=q\left(\frac{d\mu_r}{dq}+\frac{rk}{p^2}\mu_{r-1}\right),\ r=1,2,\dots\dots$$

Hence find variance of X. 6

6. (a) If X and Y are independent Poisson variates such that

$V(X+Y)=4$ and $P(X=3\mid X+Y=6)=5/16$,

then find E(X) and V(Y). 6

(b) If $X \sim N(\mu,\sigma^2)$, then find the p.d.f. of $Y = e^x$ and identify it. Also find the coefficient of variation of Y. 6

7. (a) If $X \sim N(0, 1)$ and $Y \sim N(0, 1)$ are independent random variables, then find the mean deviation about mean of $(X-Y)$. 6

(b) Let $X \sim \beta_1(m, n)$ and $Y \sim \gamma(m+n)$ be independent random variables, $(m, n>0)$. Find the p.d.f. of XY and identify the distribution. 6

8. (a) Define hypergeometric distribution and find its mean. Obtain binomial distribution as a limiting case of hypergeometric distribution. 6

(b) $X_1, X_2, \dots\dots X_n$ are independent random variable having exponential distribution each with parameter λ. Obtain the distribution of $Y = X_1 + X_2 + \dots\dots + X_n$ and hence find mean of Y. 6